Problems 5 and 6 are for homework. Solutions must be submitted before the next lecture (not tutorial!) to be evaluated. Students are not allowed to keep submitted solutions after evaluation.

Problem 1. Consider the polyhedron P given by the following conditions.

$$\begin{array}{rcrcl} x_1 & - & x_2 & \leq & -1 \\ -x_1 & - & x_2 & \leq & -3 \\ 2x_1 & - & x_2 & \leq & 2 \\ x_1, x_2 \geq 0. & & & & \end{array}$$

1. Draw the polyhedron P.
2. Using the Simplex methods find all vertices of P.
3. Find the optimal solution of the problem $\min\{x_1 + 2x_2;\ x \in P\}$.
4. Find the optimal solution of the problem $\max\{3x_1 + x_2;\ x \in P\}$.

Problem 2. Solve the following problem

$$\begin{array}{lrcrcl} \text{Maximize} & 3x_1 & + & x_2 & & \\ \text{subject to} & x_1 & - & x_2 & \leq & -1 \\ & -x_1 & - & x_2 & \leq & -3 \\ & 2x_1 & + & x_2 & \leq & 2 \\ & x_1, x_2 \geq 0 & & & & \end{array}$$

Problem 3. Solve the following problem

$$\begin{array}{lrcrcrcl} \text{Minimize} & -2x_1 & + & 4x_2 & - & x_3 & & \\ \text{subject to} & 3x_1 & - & 6x_2 & + & 4x_3 & \leq & 30 \\ & 2x_1 & - & 8x_2 & + & 10x_3 & \geq & 18 \\ & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

Problem 4. Solve the following linear programming problem.

$$\begin{array}{lrcrcrcl} \text{Maximize} & 4x & + & 5y & + & 3z & & \\ \text{subject to} & x & + & y & + & 2z & \geq & 20 \\ & 5x & + & 6y & + & 5z & \leq & 50 \\ & x & + & 3y & + & 5z & \leq & 30 \\ & & & & & x, y, z & \geq & 0 \end{array}$$

Problem 5. Find all optimal vertices of the following problem.

$$\begin{array}{lrcrcrcrcl} \text{Maximize} & 2\,x_1 & + & 3\,x_2 & + & 5\,x_3 & + & 4\,x_4 & & \\ \text{subject to} & x_1 & + & 2\,x_2 & + & 3\,x_3 & + & x_4 & \leq & 5 \\ & x_1 & + & x_2 & + & 2\,x_3 & + & 3\,x_4 & \leq & 3 \\ & x_1 & + & x_2 & + & 2\,x_3 & + & 7\,x_4 & \geq & 3 \\ & & & & & & x_1, x_2, x_3, x_4 & & \geq & 0 \end{array}$$

Problem 6. Solve the following problem

$$\begin{array}{lrcrcrcrcrcl} \text{Maximize} & & & & 10\,x_1 & - & 57\,x_2 & - & 9\,x_3 & - & 24\,x_4 \\ \text{subject to} & x_5 & = & - & 0{,}5\,x_1 & + & 5{,}5\,x_2 & + & 2{,}5\,x_3 & - & 9\,x_4 \\ & x_6 & = & - & 0{,}5\,x_1 & + & 1{,}5\,x_2 & + & 0{,}5\,x_3 & - & x_4 \\ & x_7 & = & 1 - & x_1 & & & & & & \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 & & & & & & & & & \end{array}$$

First, try to use the pivot rule "largest coefficient". Then, solve the problem using "Bland rule".