The sixth homework are Problems 2 and 6.

Problem 1. Find all vertices and edges of a polyhedron determined by the following conditions.

$$
\begin{aligned}
x_1 + x_2 + x_3 &\le 8 \\
x_1 &\le 6 \\
x_2 + x_3 &\le 4 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

Problem 2. Find all vertices and edges of a polyhedron determined by the following conditions.

$$
\begin{aligned}
2x_1 + x_2 + x_3 &\le 14 \\
2x_1 + 5x_2 + 5x_3 &\le 30 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

Problem 3. Solve the following problem

$$
\begin{aligned}
\text{Maximize} \quad & x_1 + 2x_2 \\
\text{subject to} \quad & x_1 + 3x_2 + x_3 = 4 \\
& 2x_2 + x_3 = 2 \\
& x_1, x_2, x_3 \ge 0.
\end{aligned}
$$

Problem 4. Solve the following problem

$$
\begin{aligned}
\text{Maximize} \quad & x_1 + 2x_2 \\
\text{subject to} \quad & x_1 - x_2 \le 2 \\
& -x_1 + x_2 \le 1 \\
& 2x_1 + x_2 \le 7 \\
& x_1, x_2 \ge 0.
\end{aligned}
$$

Problem 5. Solve the following problem

$$
\begin{aligned}
\text{Maximize} \quad & x_1 + 2x_2 - x_3 \\
\text{subject to} \quad & 2x_1 + x_2 + x_3 \le 14 \\
& 4x_1 + 2x_2 + 3x_3 \le 28 \\
& 2x_1 + 5x_2 + 5x_3 \le 30 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

Problem 6. Solve the following problem

$$
\begin{aligned}
\text{Maximize} \quad & 2x_1 - x_2 + 2x_3 \\
\text{subject to} \quad & 2x_1 + x_2 \le 10 \\
& x_1 + 2x_2 - 2x_3 \le 20 \\
& x_2 + 2x_3 \le 5 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$