The seventh homework are Problems 5 and 6.

Problem 1. Consider the polyhedron P given by the following conditions.

$$\begin{array}{rcrcl} x_1 & - & x_2 & \leq & -1 \\ -x_1 & - & x_2 & \leq & -3 \\ 2x_1 & - & x_2 & \leq & 2 \\ x_1, x_2 \geq 0. & & & & \end{array}$$

1. Draw the polyhedron P.
2. Using the Simplex methods find all vertices of P.
3. Find the optimal solution of the problem $\min\{x_1 + 2x_2;\ x \in P\}$.
4. Find the optimal solution of the problem $\max\{3x_1 + x_2;\ x \in P\}$.

Problem 2. Solve the following problem

$$\begin{array}{lrcrcl} \text{Maximize} & 3x_1 & + & x_2 & & \\ \text{subject to} & x_1 & - & x_2 & \leq & -1 \\ & -x_1 & - & x_2 & \leq & -3 \\ & 2x_1 & + & x_2 & \leq & 2 \\ & x_1, x_2 \geq 0 & & & & \end{array}$$

Problem 3. Solve the following problem

$$\begin{array}{lrcrcrcl} \text{Minimize} & -2x_1 & + & 4x_2 & - & x_3 & & \\ \text{subject to} & 3x_1 & - & 6x_2 & + & 4x_3 & \leq & 30 \\ & 2x_1 & - & 8x_2 & + & 10x_3 & \geq & 18 \\ & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

Problem 4. Solve the following linear programming problem.

$$\begin{array}{lrcrcrcl} \text{Maximize} & 4x & + & 5y & + & 3z & & \\ \text{subject to} & x & + & y & + & 2z & \geq & 20 \\ & 5x & + & 6y & + & 5z & \leq & 50 \\ & x & + & 3y & + & 5z & \leq & 30 \\ & & & & & x, y, z & \geq & 0 \end{array}$$

Problem 5. Find all optimal vertices of the following problem.

$$\begin{array}{lrcrcrcrcl} \text{Maximize} & 2\,x_1 & + & 3\,x_2 & + & 5\,x_3 & + & 4\,x_4 & & \\ \text{subject to} & x_1 & + & 2\,x_2 & + & 3\,x_3 & + & x_4 & \leq & 5 \\ & x_1 & + & x_2 & + & 2\,x_3 & + & 3\,x_4 & \leq & 3 \\ & x_1 & + & x_2 & + & 2\,x_3 & + & 7\,x_4 & \geq & 3 \\ & & & & & & x_1, x_2, x_3, x_4 & & \geq & 0 \end{array}$$

Problem 6. Solve the following problem

$$\begin{array}{lrcrcrcrcrcl} \text{Maximize} & & & & 10\,x_1 & & - & 57\,x_2 & - & 9\,x_3 & - & 24\,x_4 \\ \text{subject to} & x_5 & = & - & 0{,}5\,x_1 & & + & 5{,}5\,x_2 & + & 2{,}5\,x_3 & - & 9\,x_4 \\ & x_6 & = & - & 0{,}5\,x_1 & & + & 1{,}5\,x_2 & + & 0{,}5\,x_3 & - & x_4 \\ & x_7 & = & 1 - & x_1 & & & & & & & \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 & & & & & & & & & & \end{array}$$

First, try to use the pivot rule "largest coefficient". Then, solve the problem using "Bland rule".