The eight homework are Problems 4 and 5.

Every linear programming problem has its dual, e.g.
- Maximize $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ subject to $A\boldsymbol{x}\geq\boldsymbol{b}$ and $\boldsymbol{x}\geq\boldsymbol{0}$ — Primal program
- Maximize $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ subject to $-A\boldsymbol{x}\leq-\boldsymbol{b}$ and $\boldsymbol{x}\geq\boldsymbol{0}$ — Equivalent formulation
- Minimize $-\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}$ subject to $-A^{\mathrm{T}}\boldsymbol{y}\geq\boldsymbol{c}$ and $\boldsymbol{y}\geq\boldsymbol{0}$ — Dual program
- Minimize $\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}$ subject to $A^{\mathrm{T}}\boldsymbol{y}\geq\boldsymbol{c}$ and $\boldsymbol{y}\leq\boldsymbol{0}$ — Simplified formulation

A dual of a dual problem is the (original) primal problem, e.g.
- Minimize $\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}$ subject to $A^{\mathrm{T}}\boldsymbol{y}\geq\boldsymbol{c}$ and $\boldsymbol{y}\geq\boldsymbol{0}$ — Dual program
- -Maximize $-\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}$ subject to $A^{\mathrm{T}}\boldsymbol{y}\geq\boldsymbol{c}$ and $\boldsymbol{y}\geq\boldsymbol{0}$ — Equivalent formulation
- -Minimize $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ subject to $A\boldsymbol{x}\geq-\boldsymbol{b}$ and $\boldsymbol{x}\leq\boldsymbol{0}$ — Dual of the dual program
- -Minimize $-\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ subject to $-A\boldsymbol{x}\geq-\boldsymbol{b}$ and $\boldsymbol{x}\geq\boldsymbol{0}$ — Simplified formulation
- Maximize $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ subject to $A\boldsymbol{x}\leq\boldsymbol{b}$ and $\boldsymbol{x}\geq\boldsymbol{0}$ — The original primal program

General rules:

	Primal linear program	Dual linear program
Variables	$\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n$	$\boldsymbol{y}_1,\ldots,\boldsymbol{y}_m$
Matrix	A	A^{T}
Right-hand side	$\boldsymbol{b}$	$\boldsymbol{c}$
Objective function	$\max\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$	$\min\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}$
Constraints	i-the constraint has $\leq$	$\boldsymbol{y}_i\geq 0$
	i-the constraint has $\geq$	$\boldsymbol{y}_i\leq 0$
	i-the constraint has $=$	$\boldsymbol{y}_i\in\mathbb{R}$
	$\boldsymbol{x}_j\geq 0$	j-th constraint has $\geq$
	$\boldsymbol{x}_j\leq 0$	j-th constraint has $\leq$
	$\boldsymbol{x}_j\in\mathbb{R}$	j-th constraint has $=$

Problem 1. Find the dual problem to the following linear programming problems and write the complementary slackness conditions.

1. $\max c^T x$ subject to $Ax\leq b$,
2. $\max c^T x$ subject to $Ax=b, x\geq 0$,
3. $\min c^T x$ subject to $A_1x=b_1, A_2x\geq b_2$.

Problem 2. Find the dual problem to the following linear programming problem and write the complementary slackness conditions.

$$\begin{array}{lrcrcrcr}
\text{Minimize} & 2x_1 & & & - & 2x_3 & & \\
\text{subject to} & 7x_1 & + & 10x_2 & + & 2x_3 & \leq & 23 \\
& 2x_1 & + & 3x_2 & + & x_3 & = & 5 \\
& -4x_1 & + & 14x_2 & - & 3x_3 & \geq & 11 \\
& & & & & x_1 & \geq & 0 \\
& & & & & x_3 & \leq & 0
\end{array}$$

Problem 3. Find the dual problem to the following two linear programming problems and write the complementary slackness conditions.

$$
\begin{array}{rrrrrrrrrl}
\text{1) Maximize} & x_1 & - & 2x_2 & & & + & 3x_4 & & \\
\text{2) Minimize} & x_1 & - & 2x_2 & & & + & 3x_4 & & \\
\text{subject to} & & & x_2 & - & 6x_3 & + & x_4 & \leq & 4 \\
& -x_1 & + & 3x_2 & - & 3x_3 & & & = & 0 \\
& 6x_1 & - & 2x_2 & + & 2x_3 & - & 4x_4 & \geq & 5 \\
& \multicolumn{9}{l}{x_2 \leq 0,\ x_4 \geq 0}
\end{array}
$$

Problem 4. Write the dual problem and the complementary slackness conditions for the following linear programming problem.

$$
\begin{array}{lrcrcrcr}
\text{Minimize} & 2x_1 & + & 3x_2 & - & 4x_3 & & \\
\text{subject to} & 6x_1 & - & 7x_2 & + & 8x_3 & = & 9 \\
& -10x_1 & + & 11x_2 & - & 12x_3 & \geq & 13 \\
& 14x_1 & - & 15x_2 & + & 16x_3 & \leq & 17 \\
& & & & & x_1 & \geq & 0 \\
& & & & & x_2 & \leq & 0 \\
& & & & & x_3 & \in & \mathbb{R}
\end{array}
$$

Problem 5. Write an integer linear programming problem for minimal weight perfect matching problem of a graph. Consider the relaxation of that matching problem (i.e. ignore all integral conditions). Find the dual problem of this relaxed problem and write the complementary slackness conditions (for the relaxed and the dual problem).