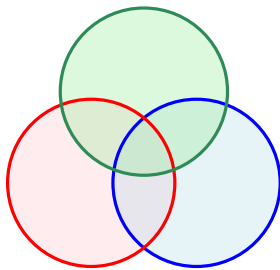


Almost simple symmetric Venn diagrams

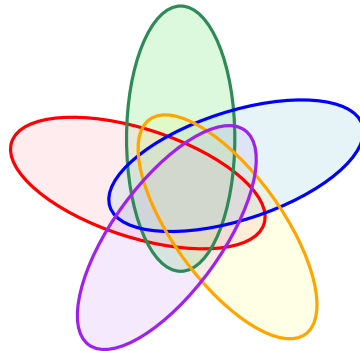
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A n -Venn diagram is *simple* if in every crossing only two curves intersect, and it is *symmetric* if it has an n -fold rotational symmetry around some point that maps each curve to the next curve. Is there a simple symmetric Venn diagram for every prime n ?

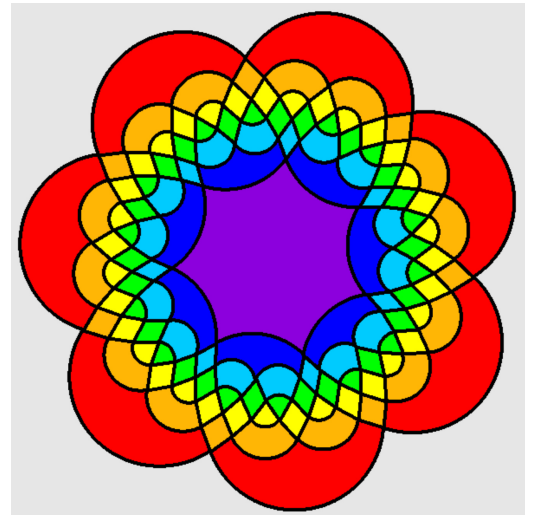
$n = 3$



$n = 5$



$n = 7$



$n = 11, 13$ ✓

$n = 17, 19, \dots$?

This open question seems hard, but there are known constructions of *non-simple* symmetric Venn diagrams for every prime n where more than two curves may intersect in each crossing. The best known construction achieves asymptotically 2^{n-1} crossings [3].

The goal of the thesis is to find better constructions of symmetric Venn diagrams with larger number of crossings, as close as to $2^n - 2$, which is attained (exactly) by simple n -Venn diagrams. This could be done through symmetric chain decompositions of the necklace poset and finding a dual of a Venn diagram with large number of faces on carefully selected necklace representatives.

So, this is essentially a combinatorial, not only geometric topic.

References

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