

Midterm test

Nov 19, 2025

An interrogation of three suspects Adam, Barbara and Cyril revealed the following facts:

- (i) At least one of them speaks the truth and at least one of them lies.
 - (ii) Adam says: “Barbara or Cyril lies.”
 - (iii) Barbara says: “Cyril lies.”
 - (iv) Cyril says: “Adam or Barbara lies.”
1. Express the facts (i) to (iv) as propositions φ_1 to φ_4 over $\mathbb{P} = \{a, b, c\}$ where a, b, c means (respectively) that “Adam / Barbara / Cyril speaks the truth”. (10 pts)
 2. Let $T = \{\varphi_1, \dots, \varphi_4\}$. Axiomatize $M(T)$ by a proposition in CNF and by a proposition in DNF. (20 pts)
 3. Prove by the tableau method that $T \models a$, i.e., from these facts it follows that “Adam speaks the truth”. Use only the original propositions in the tableau and not their CNF or DNF equivalent forms. (25 pts)
 4. Prove that $T \models a$ by the resolution method. Draw the proof as a resolution tree. (25 pts)
 5. How many there exist nonequivalent propositions over $\mathbb{P}$ that are valid in T and (at the same time) not valid in $S = \{\varphi_3\}$? Explain how you obtained this number. (20 pts)