Final test

Jan 8, 2026

1. Let T be the following theory in the language $L = \langle P \rangle$ without equality, where P is a binary relation symbol:

$$
\begin{aligned}
T = \{&\neg(\exists x)P(x,x),\\
&(\exists x)(\forall y)(P(x,y) \rightarrow P(y,x)),\\
&(\forall x)((\exists y)(P(x,y) \wedge P(y,x)) \rightarrow (\exists y)P(y,y)),\\
&(\forall x)(\exists y)P(x,y)\}
\end{aligned}
$$

   (a) Applying skolemization find a theory T' (in a some extended language) such that T' is equisatisfiable with T and all axioms of T' are universal sentences. *(20 pts)*

   (b) Prove by tableau method that T' is unsatisfiable. *(30 pts)*

   (c) Let T'' denote the set of open matrices of axioms of T' (obtained by removing the prefix of quantifiers). Use resolution to show that T'' is not satisfiable. Express the refutation as a resolution tree. In each step write the unification used. *(30 pts)*

   (d) Find a conjunction of ground instances of axioms of T'' that is unsatisfiable. *(10 pts)*

   (e) Is the sentence $\neg(\exists y)P(y,y)$ valid in $S = T \setminus \{\neg(\exists x)P(x,x)\}$? Is it contradictory in S? Is it independent in S? Justify all three answers. *(10 pts)*