

Predicate and Propositional Logic - Tutorial 4

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1. Applying the implication graph determine whether the following proposition in 2-CNF is satisfiable or not; and if yes, find a satisfying assignment.

$$(p_0 \vee p_2) \wedge (p_0 \vee \neg p_3) \wedge (p_1 \vee \neg p_3) \wedge (p_1 \vee \neg p_4) \wedge (p_2 \vee \neg p_4) \wedge (p_0 \vee \neg p_5) \wedge \\ (p_1 \vee \neg p_5) \wedge (p_2 \vee \neg p_5) \wedge (\neg p_1 \vee \neg p_6) \wedge (p_4 \vee p_6) \wedge (p_5 \vee p_6) \wedge p_1$$

2. Applying unit propagation determine whether the following Horn formula is satisfiable; and if yes, find a satisfying assignment.

$$(\neg p_1 \vee \neg p_3 \vee p_2) \wedge (\neg p_1 \vee p_2) \wedge p_1 \wedge (\neg p_1 \vee \neg p_2 \vee p_3) \wedge \\ (\neg p_2 \vee \neg p_4 \vee p_1) \wedge (p_4 \vee \neg p_3 \vee \neg p_2) \wedge (\neg p_4 \vee p_5)$$

3. Find tableau proofs of the following tautologies.

- (a) $(p \rightarrow (q \rightarrow q))$
- (b) $p \leftrightarrow \neg \neg p$
- (c) $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$
- (d) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- (e) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

Are the constructed tableaux systematic?

4. Applying tableau method prove the following propositions or find counterexamples

- (a) $\{\neg q, p \vee q\} \models p$,
- (b) $\{q \rightarrow p, r \rightarrow q, (r \rightarrow p) \rightarrow s\} \models s$,
- (c) $\{p \rightarrow r, p \vee q, \neg s \rightarrow \neg q\} \models r \rightarrow s$.

5. Applying tableau method determine all models of the following theories.

- (a) $\{(\neg p \vee q) \rightarrow (\neg q \wedge r)\}$
- (b) $\{\neg q \rightarrow (\neg p \vee q), \neg p \rightarrow q, r \rightarrow q\}$
- (c) $\{q \rightarrow p, r \rightarrow q, (r \rightarrow p) \rightarrow s\}$

6. Propose suitable atomic tableaux for Peirce arrow $\downarrow$ (NOR) and for Sheffer stroke $\uparrow$ (NAND).

7. Prove directly (by tableau transformations) the deduction theorem, i.e. for every theory T and propositions φ, ψ ,

$$T \vdash \varphi \rightarrow \psi \text{ if and only if } T, \varphi \vdash \psi.$$

8. Let $\mathcal{S}$ be a countable nonempty family of nonempty finite sets. We say that $\mathcal{S}$ has a *selector* if there exists an injective $f: \mathcal{S} \rightarrow \bigcup \mathcal{S}$ such that $f(S) \in S$ for every $S \in \mathcal{S}$. Prove that $\mathcal{S}$ has a selector if and only if every nonempty finite part of $\mathcal{S}$ has a selector.