Predicate and Propositional Logic - Tutorial 6

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1. Let $\mathcal{S}$ be a countable nonempty family of nonempty finite sets. We say that $\mathcal{S}$ has a *selector* if there exists an injective $f\colon \mathcal{S} \to \bigcup \mathcal{S}$ such that $f(S) \in S$ for every $S \in \mathcal{S}$. Prove that $\mathcal{S}$ has a selector if and only if every nonempty finite part of $\mathcal{S}$ has a selector.

2. Let φ be the proposition $\neg(p \vee q) \to (\neg p \wedge \neg q)$.

   (a) Transform $\neg\varphi$ into CNF and into set representation (clausal form).

   (b) Find a resolution refutation of $\neg\varphi$; that is, a proof of φ.

3. Find resolution closures $\mathcal{R}(S)$ of the following formulas S.

   (a) $\{\{p, q\}, \{\neg p, \neg q\}, \{\neg p, q\}\}$

   (b) $\{\{p, q\}, \{p, \neg q\}, \{p, \neg q\}\}$

   (c) $\{\{p, \neg q, r\}, \{q, r\}, \{\neg p, r\}, \{q, \neg r\}, \{\neg q\}\}$

4. Find resolution refutations of the following propositions.

   (a) $(p \leftrightarrow (q \to r)) \wedge ((p \leftrightarrow q) \wedge (p \leftrightarrow \neg r))$

   (b) $\neg(((p \to q) \to \neg q) \to \neg q)$

5. Prove by resolution that s is valid in a theory $T = \{\neg p \to \neg q, \neg q \to \neg r, (r \to p) \to s\}$.

6. Show that if $S = \{C_1, C_2\}$ is satisfiable and C is a resolvent of C_1 and C_2, then C is satisfiable as well.

7. Find the *tree of reductions* of a formula $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$.

8. Assume that we have available MgO, H_2, O_2, C and we can perform the following chemical reactions.

$$
\begin{array}{ll}
(1) & \mathrm{MgO} + \mathrm{H_2} \;\to\; \mathrm{Mg} + \mathrm{H_2O} \\
(2) & \mathrm{C} + \mathrm{O_2} \;\to\; \mathrm{CO_2} \\
(3) & \mathrm{CO_2} + \mathrm{H_2O} \;\to\; \mathrm{H_2CO_3}
\end{array}
$$

   (a) Represent the state of affairs as a proposition in a suitable language and transform it into a set representation.

   (b) Prove by resolution that we can produce H_2CO_3.