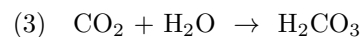
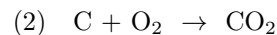
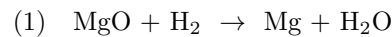


## Predicate and Propositional Logic - Tutorial 6

Nov 5, 2025

1. Let  $\mathcal{S}$  be a countable nonempty family of nonempty finite sets. We say that  $\mathcal{S}$  has a *selector* if there exists an injective  $f: \mathcal{S} \rightarrow \bigcup \mathcal{S}$  such that  $f(S) \in S$  for every  $S \in \mathcal{S}$ . Prove that  $\mathcal{S}$  has a selector if and only if every nonempty finite part of  $\mathcal{S}$  has a selector.
2. Let  $\varphi$  be the proposition  $\neg(p \vee q) \rightarrow (\neg p \wedge \neg q)$ .
  - (a) Transform  $\neg\varphi$  into CNF and into set representation (clausal form).
  - (b) Find a resolution refutation of  $\neg\varphi$ ; that is, a proof of  $\varphi$ .
3. Find resolution closures  $\mathcal{R}(S)$  of the following formulas  $S$ .
  - (a)  $\{\{p, q\}, \{\neg p, \neg q\}, \{\neg p, q\}\}$
  - (b)  $\{\{p, q\}, \{p, \neg q\}, \{p, \neg q\}\}$
  - (c)  $\{\{p, \neg q, r\}, \{q, r\}, \{\neg p, r\}, \{q, \neg r\}, \{\neg q\}\}$
4. Find resolution refutations of the following propositions.
  - (a)  $(p \leftrightarrow (q \rightarrow r)) \wedge ((p \leftrightarrow q) \wedge (p \leftrightarrow \neg r))$
  - (b)  $\neg(((p \rightarrow q) \rightarrow \neg q) \rightarrow \neg q)$
5. Prove by resolution that  $s$  is valid in a theory  $T = \{\neg p \rightarrow \neg q, \neg q \rightarrow \neg r, (r \rightarrow p) \rightarrow s\}$ .
6. Show that if  $S = \{C_1, C_2\}$  is satisfiable and  $C$  is a resolvent of  $C_1$  and  $C_2$ , then  $C$  is satisfiable as well.
7. Find the *tree of reductions* of a formula  $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$ .
8. Assume that we have available MgO, H<sub>2</sub>, O<sub>2</sub>, C and we can perform the following chemical reactions.



- (a) Represent the state of affairs as a proposition in a suitable language and transform it into a set representation.
- (b) Prove by resolution that we can produce H<sub>2</sub>CO<sub>3</sub>.