

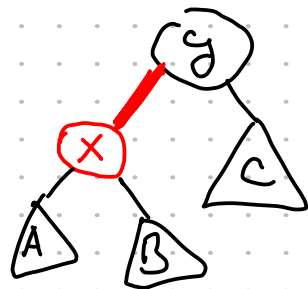
# Splay trees

- self-adjusting BST
- balancing via splay operation

## Splay(x):

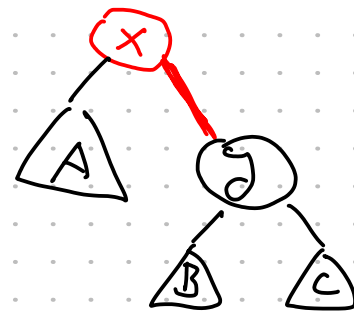
- move to root using rotations
  - use double rotations if possible
- $\Rightarrow \leq 1$  simple rot.  
(at the root)

## Rotations

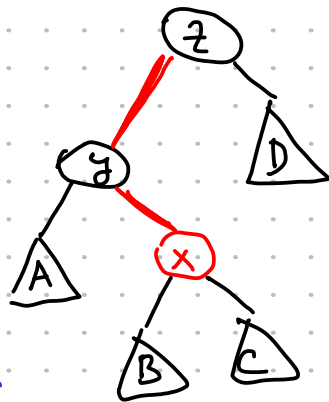


zig  $\rightarrow$

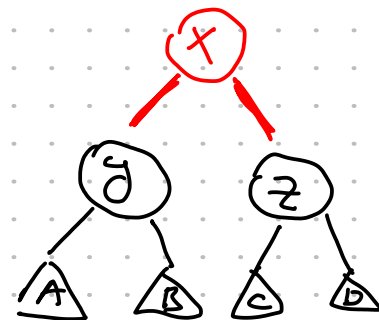
$\leftarrow$  zag



simple rotation



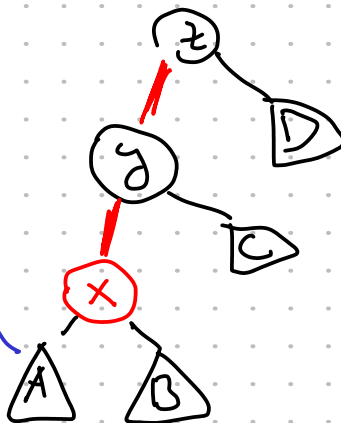
zig-zag  $\rightarrow$



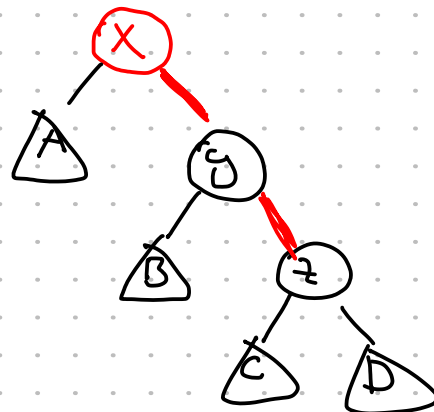
double rotation

zig(x,y)  
then zag(x,z)

zig(y,z)  
then zig(x,y)



zig-zig  $\rightarrow$



# Operations:

- always splay the deepest node operation touches
- up to  $\Theta(n)$  worst case complexity (tree can be path)
- splaying ensures amortized  $O(\log n)$  (analysis at the lecture)
- ops work like "ordinary" BST only with the splay at the end
  - another way is to first splay and then use split and join (easier analysis)

## Find(x)

- search for  $x$  (like ordinary BST)
- then splay  $x$  (if found)  
or last visited node (if not found)

## Insert(x)

- search for  $x$
- if not found insert under last visited node
- splay( $x$ )
  - ! even if found in search !



## Delete(x)

- search for  $x$
- $x$  has  $\leq 1$  children  $\Rightarrow$  remove  $x$  (like in ordinary BST)
- otherwise find successor, swap it with  $x$  and remove  $x$  (it has  $\leq 1$  children now)
- splay parent of removed node  
or last visited node if  $x$  not found